Name of College - S.S. College, J-Bad
Deptt - Mathematics
Topic - Problem on plane (contd)
(Analytical Geometry of 3-D)
Class - B.Sc I (HONS)

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Time - 1:00 P.M To 1:45 PM
Date - 07-08-2020 By - Dr. Shree Nath Sharma

Problem :→ Find the Equation of plane through the point (2, -1, 1) and the line of intersection of the given planes $4x - 3y + 5 = 0$

$y - 2z - 5 = 0$

Here Given planes are

$$4x - 3y + 5 = 0$$

and $$y - 2z - 5 = 0$$

the Equation of plane passing through the line of intersection of given planes is

$$(4x - 3y + 5) + \lambda(y - 2z - 5) = 0 \quad \text{---(1)}$$

$$\Rightarrow 4x + (\lambda - 3)y - 2\lambda z + (5 + 5\lambda) = 0$$

Since it passes passes through the point (2, -1, 1)

Therefore $4 \times 2 + (\lambda - 3)(-1) - 2\lambda \times 1 + (5 + 5\lambda) = 0$

$$\Rightarrow 8 - \lambda + 3 - 2\lambda + 5 - 5\lambda = 0$$

$$\Rightarrow 16 - 8\lambda = 0$$

$$\lambda = 2$$

Putting $\lambda = 2$ in (1)

$$(4x - 3y + 5) + 2(y - 2z - 5) = 0$$

$$4x - y - 4z - 5 = 0$$

2. Problem :→ ~~Find~~ Prove that the Equation to the plane through (α, β, γ) and parallel to $ax + by + cz = 0$ is $ax + by + cz = a\alpha + b\beta + c\gamma$

Solution :→ By the question, given plane is $ax + by + cz = 0$ ---(1)

Therefore any plane parallel to given plane is

$$ax+by+cz=k \quad \text{——— (1)}$$

Since it passes through (α, β, γ)

therefore $a\alpha+b\beta+c\gamma=k$

putting $k=a\alpha+b\beta+c\gamma$ in (1)

we have the required plane is

$$ax+by+cz=a\alpha+b\beta+c\gamma$$

Problem – Find the Equation of the plane through $(2,3,-4)$ and $(1,-1,3)$ and Parallel to the x-axis

Solution :→

Let the Equation of plane is

$$ax+by+cz+d=0 \quad \text{——— (I)}$$

Since it passes through $(2,3,-4)$ and $(1,-1,3)$

$$\therefore \quad 2a+3b-4c+d=0 \quad \text{——— (II)}$$

and $$a-b+3c+d=0 \quad \text{——— (III)}$$

(II) – (III)

$$a+4b-7c=0 \quad \text{——— (IV)}$$

Since (I) is parallel to x-axis

Whose direction cosines are $(1,0,0)$

$$\therefore a\cdot 1+b\cdot 0+c\cdot 0=0$$

$$\Rightarrow a=0$$

putting this value in (IV)

$$0+4b-7c=0$$

$$b=\frac{7}{4}c$$

From (II) $\quad 2\cdot 0+\frac{3\times 7}{4}c-4c+d=0$

$\Rightarrow d = -\frac{5}{4}C$

Putting these values in (i)

$\frac{7}{4}Cy + Cz - \frac{5}{4}C = 0$

$7y + 4z - 5 = 0$

Problem :- Find the Equation of the plane through the points (2,2,1) and (9,3,6) and perpendicular to the plane

$2x + 6y + 6z = 9$

Solution →

By the question, the Equation of given plane is

$2x + 6y + 6z = 9$ ——— (i)

Let the Equation of plane perpendicular to given plane (i) is

$ax + by + cz + d = 0$ ——— (ii)

then $2a + 6b + 6c = 0$ ——— (iii)

By question (ii) passes through the points (2,2,1) and (9,3,6)

$\therefore 2a + 2b + c + d = 0$ ——— (iv)

$9a + 3b + 6c + d = 0$ ——— (v)

Now (v) − (iv)

$7a + b + 5c = 0$ ——— (vi)

Now By Cross multiplication of

$2a + 6b + 6c = 0$

$7a + b + 5c = 0$

$\frac{a}{30-6} = \frac{b}{42-10} = \frac{c}{2-42}$

$$\Rightarrow \frac{a}{24} = \frac{b}{32} = \frac{c}{-40}$$

$$\Rightarrow \frac{a}{3} = \frac{b}{4} = \frac{c}{-5} = -k$$

$$\Rightarrow a = 3k \quad b = 4k \quad c = -5k$$

From (4) $6k + 8k - 5k + d = 0$

$$d = -9k$$

Putting these values in (2)

we have $3kx + 4ky - 5kz - 9k = 0$

$$\Rightarrow 3x + 4y - 5z - 9 = 0$$

Which is the required Equation of Plane.

Problem :→ Find the Equation of the plane through the points $(1, -2, 4)$ and $(3, -4, 5)$ and perpendicular to yz plane.

Solution – Equation of xy plane is $z = 0$

Equation of plane parallel to xy plane is $z = k$

Equation of yz plane is $x = 0$

Equation of plane Parallel to yz plane is $x = c$

$c \neq 0$.

By the question.

Equation of the given yz Plane is

$$x = 0 \quad \text{——— (1)}$$

Now we have to Find

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Equation of plane through the points $(1,-2,4)$ and $(3,-4,5)$ and perpendicular to (1)

Let the equation of plane perpendicular to yz plane is

$$ax+by+cz+d=0 \quad \text{——— (II)}$$

Since it passes through the points $(1,-2,4)$ and $(3,-4,5)$

Therefore we have

$$a-2b+4c+d=0 \quad \text{——— (III)}$$

and $$3a-4b+5c+d=0 \quad \text{— (IV)}$$

Since (II) is perpendicular to (I)

$$\therefore a\cdot 1+b\cdot 0+c\cdot 0=0 \Rightarrow a=0$$

[If $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ are mutual perpendicular plane

$$\Rightarrow a_1a_2+b_1b_2+c_1c_2=0$$

putting $a=0$ in (III) and (IV)

$$-2b+4c+d=0$$
$$-4b+5c+d=0$$

By cross multiplication

$$\frac{b}{4-5}=\frac{c}{-4+2}=\frac{d}{-10+16}$$

$$\Rightarrow \frac{b}{-1}=\frac{c}{-2}=\frac{d}{6}=k$$

$$\Rightarrow b=-k \quad c=-2k \quad d=6k$$

putting these values in (2)

$$-ky-2kz+6k=0$$

$$\Rightarrow y+2z-6=0$$ which is the required equation of the plane.